COORDINATE SYSTEMS

Being able to describe any object's position is of great importance to a physicist. As someone familar with class 11-12 level mathematics and physics, you might intend to use the Cartesian system to describe the position of any object.

But there are other coordinate systems we can define that are more useful depending on the system we are studying.

Note that, the laws of physics are independent of the coordinate system we are using. We define coordinate systems based on our convenience to study physical laws that are imposed upon us by nature

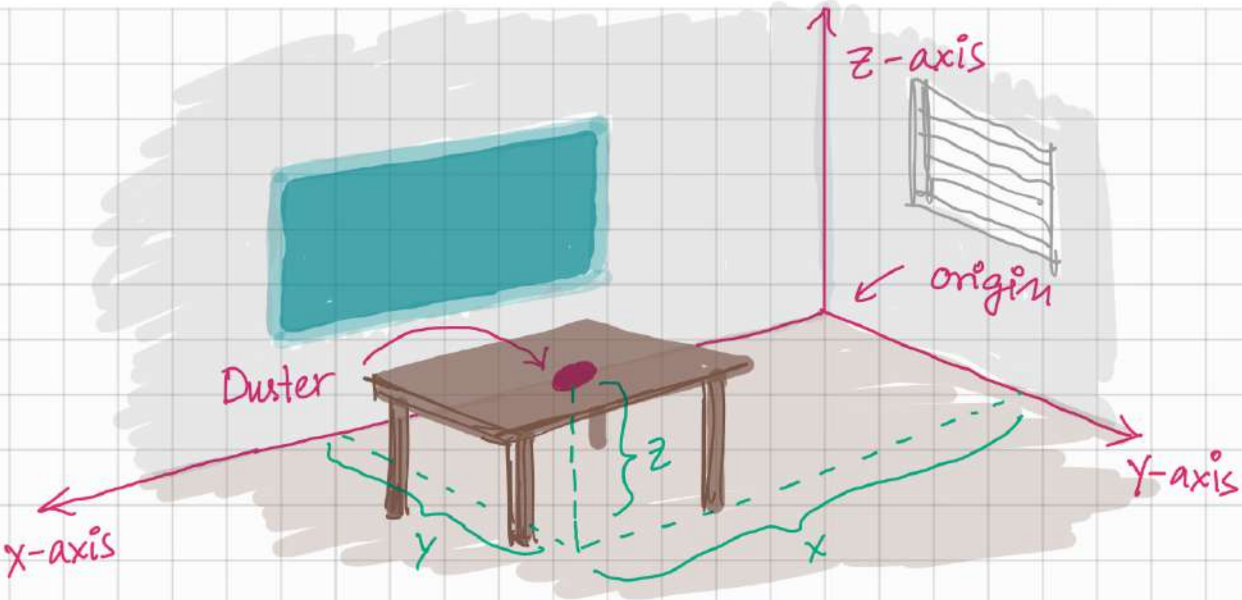
We are going to study two coordinate systems -

(i) spherical polar
(ii) cylindrical

You have a duster on the table inside your classroom. How would you describe its position?

You may just consider one corner of your class room as the origin an define the theree Cartesian axes - X-axis, Y-axis and Z-axis.

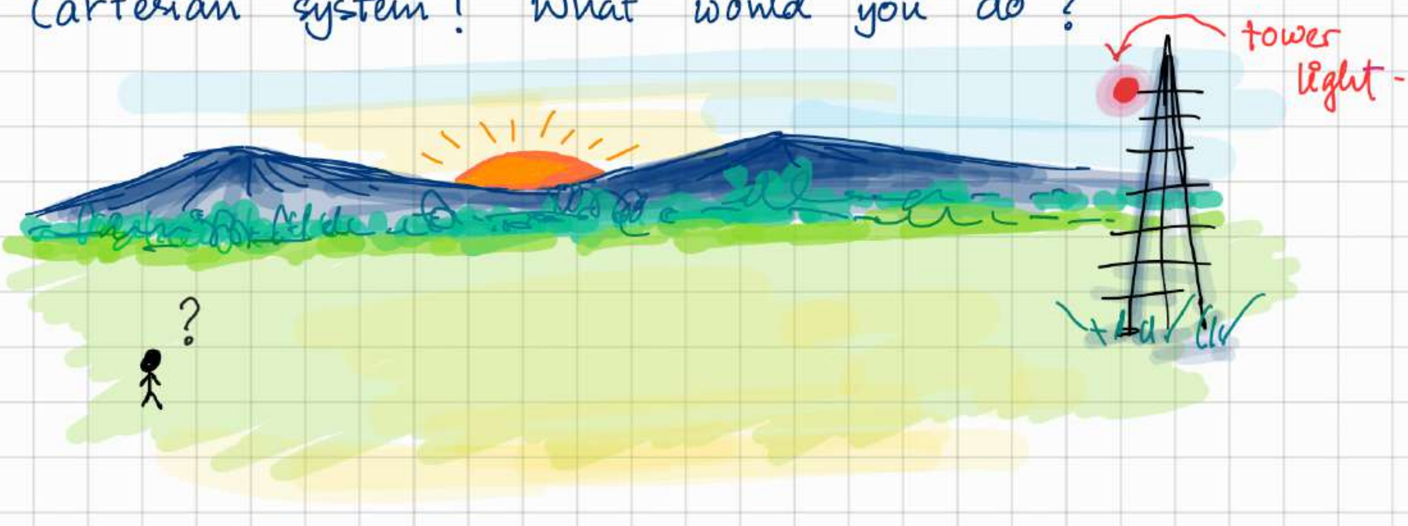
Describing the position of the duster is child's play!



SPHERICAL POLAR COORDINATE SYSTEM

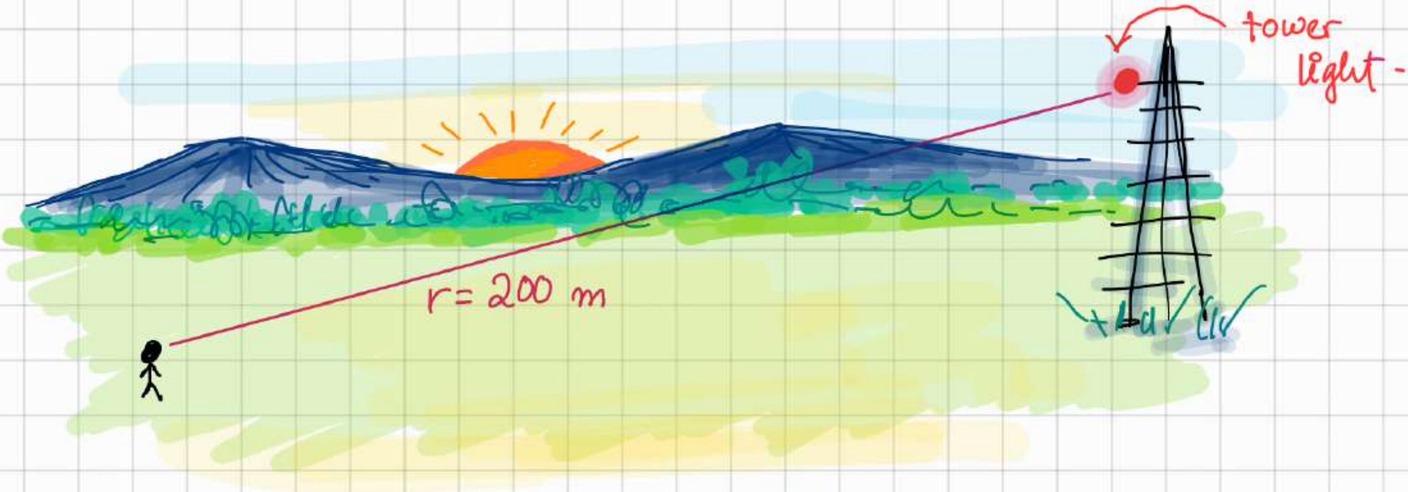
Imagine you went for a walk early morning and you stopped in the middle of a field. There is a light at the top of a mobile network tower. The sun is slowly rising above the far away mountains. You want to describe the position of the red light on the mobile tower!

Now you don't have the convenience of classroom corners in the middle of the field to use our Cartesian system! What would you do?



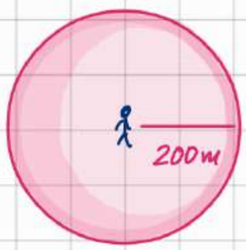
Let's think of something else.

Assume you somehow know the exact distance from your position to the light. Say it is about 200 metres.

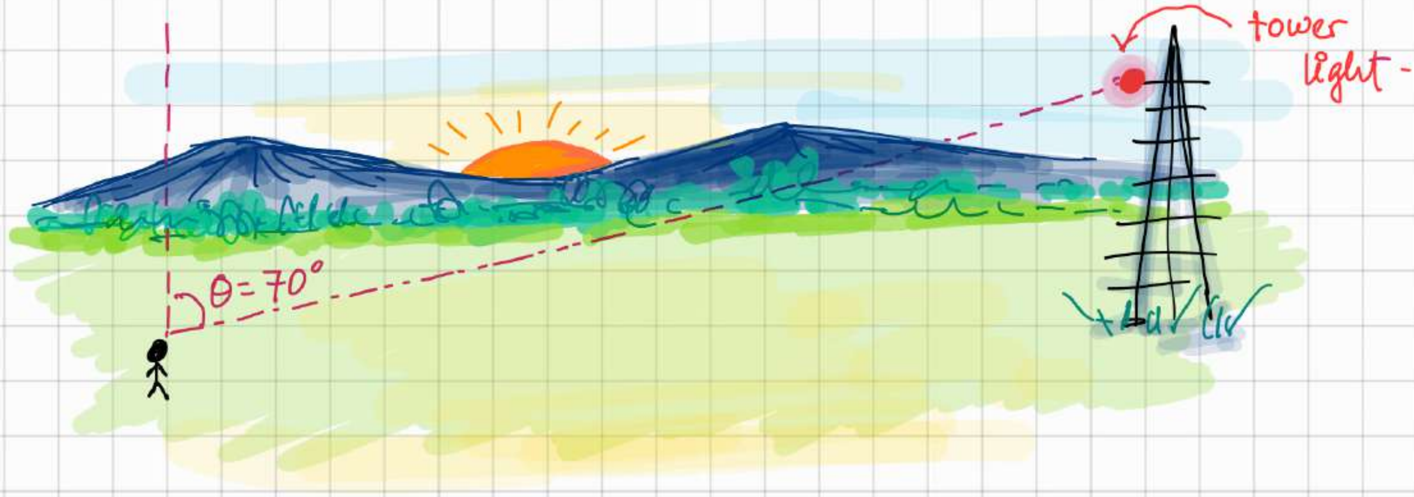


We have one information about the position of the light i.e. $r = 200$ m.

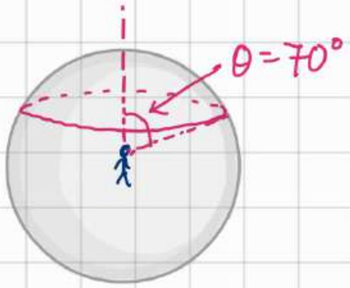
But this is not sufficient to describe the exact position of the light. the light may be to your left, to your right, straight up, straight down or any other arbitrary direction! The light may be on any point on the surface of a sphere of radius $r = 200$ m.



You might need to specify a direction now.
You decide to look straight up and call it $\theta = 0°$ and the angle to the light is, say, $\theta = 70°$.



Now you have effectively narrowed the position of the light to a circle.



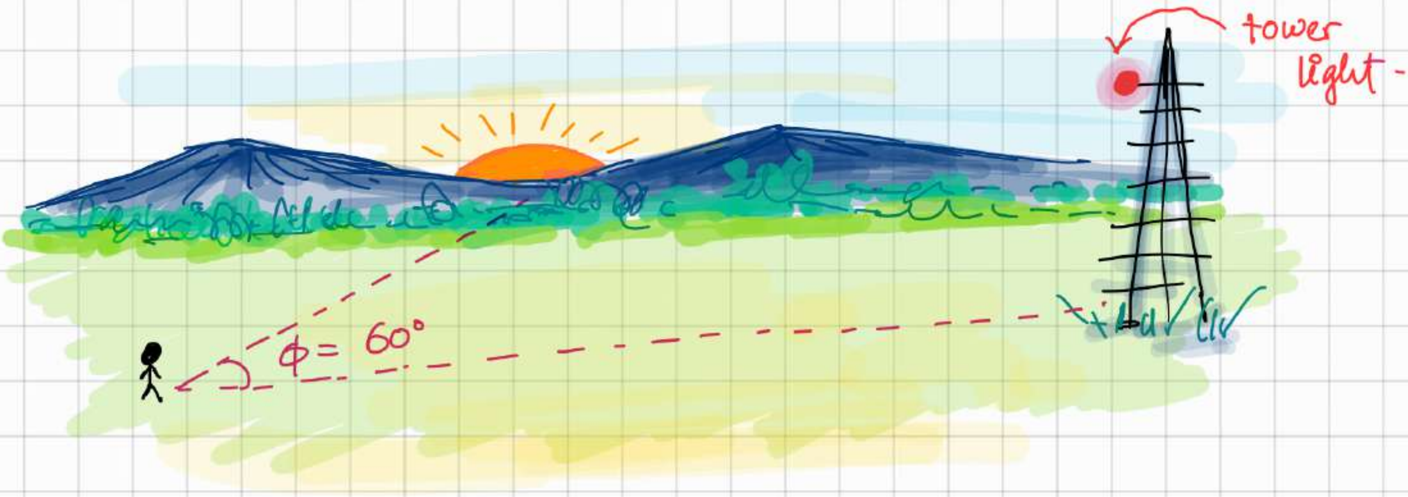
The point can be anywhere on this circle.

We still haven't yet full described the light's position. What more can we do?

Well, we can specify another direction.

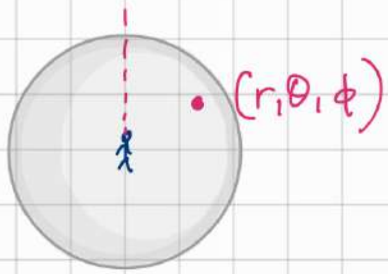
You now look at the rising sun and label this direction as $\phi = 0°$. You measure the angle on the ground from the direction of the rising sun (i.e east) to the light on the tower.

Say this $\phi = 60°$.



By specifying this direction you have just completed the full description of the position of the light relative to you (you are at the origin)

This, my friend, is a new co-ordinate system you have just developed.

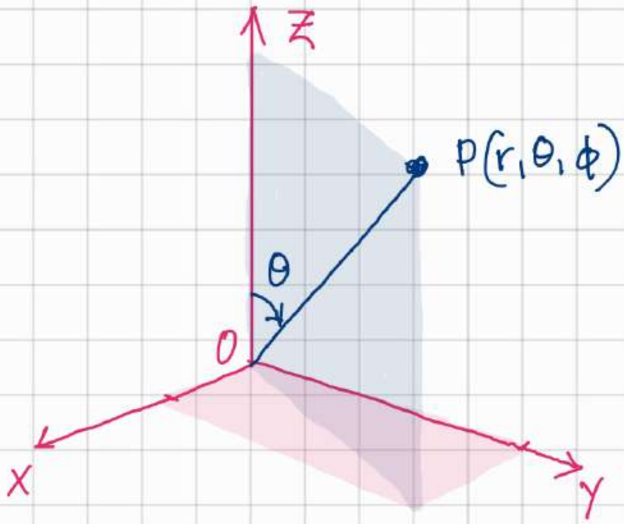


This system of specifying the position of an object using the r, θ and ϕ coordinates is nothing but the spherical polar coordinate system.

* Spherical polar co-ordinate system has three co-ordinates (r, θ, ϕ)

How are these related to the cartesian system coordinate (x, y, z) ?

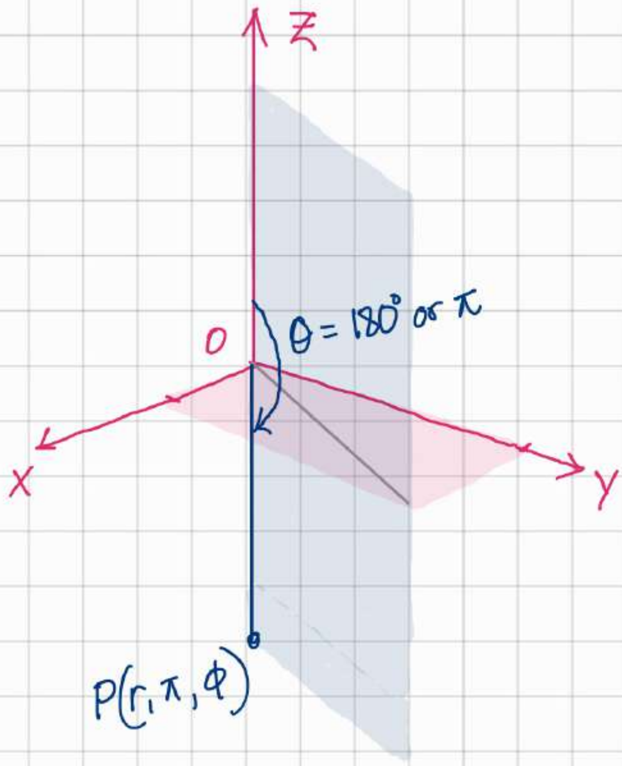
We conventionally take the vertical axis to be the z-axis in the cartesian system.



θ is measured from the positive z-axis.

So, along the z-axis, the value of θ is 0°.

What is the maximum value θ can take?

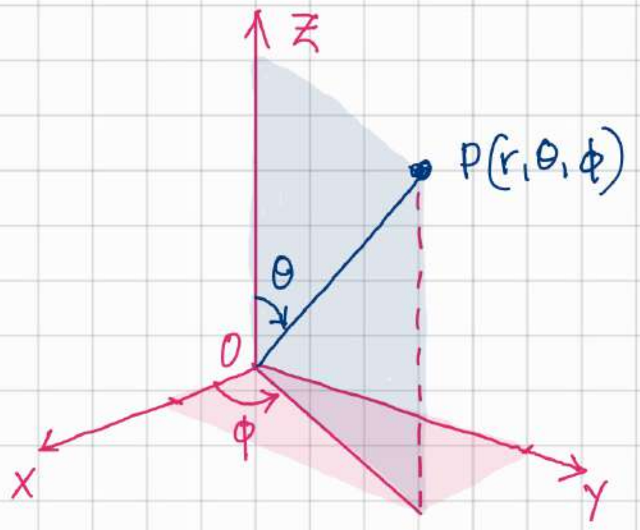


θ can assume a maximum value of π or 180°.

* In mathematical physics, it is a common practice to measure the angles in radians and not in degrees.

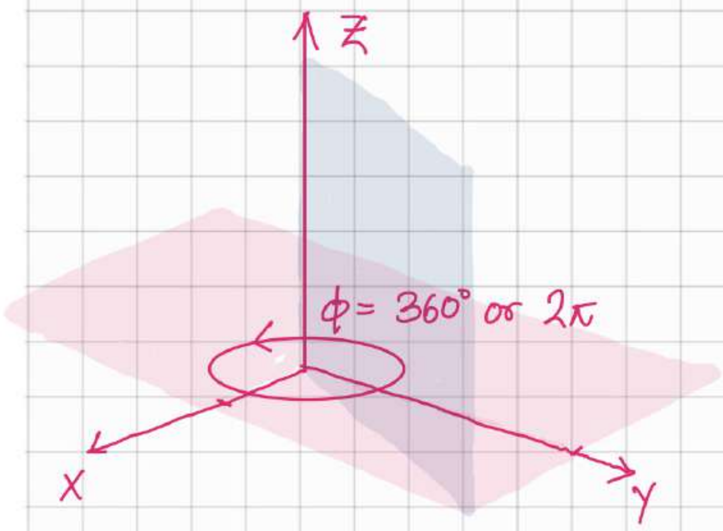
What about the ϕ co-ordinate ?

↳ It is conventional to measure the ϕ-coordinate starting from the X-axis along the X-Y plane.



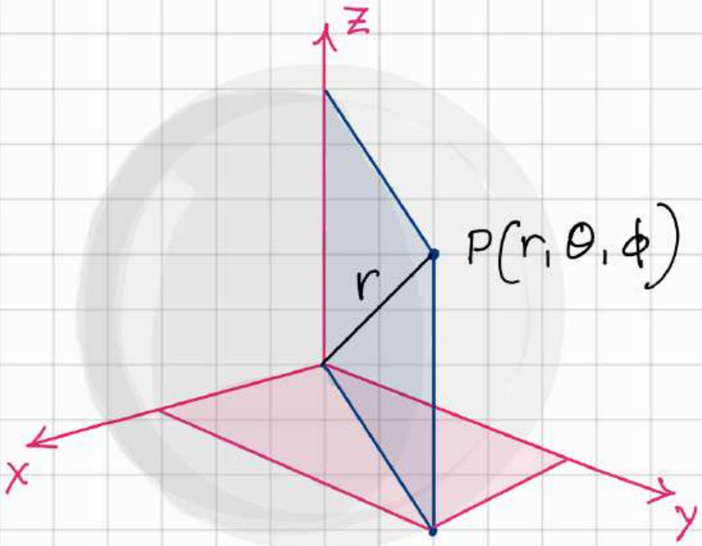
Along the direction of the X-axis, $\phi = 0$

What is the maximum value ϕ can take ?



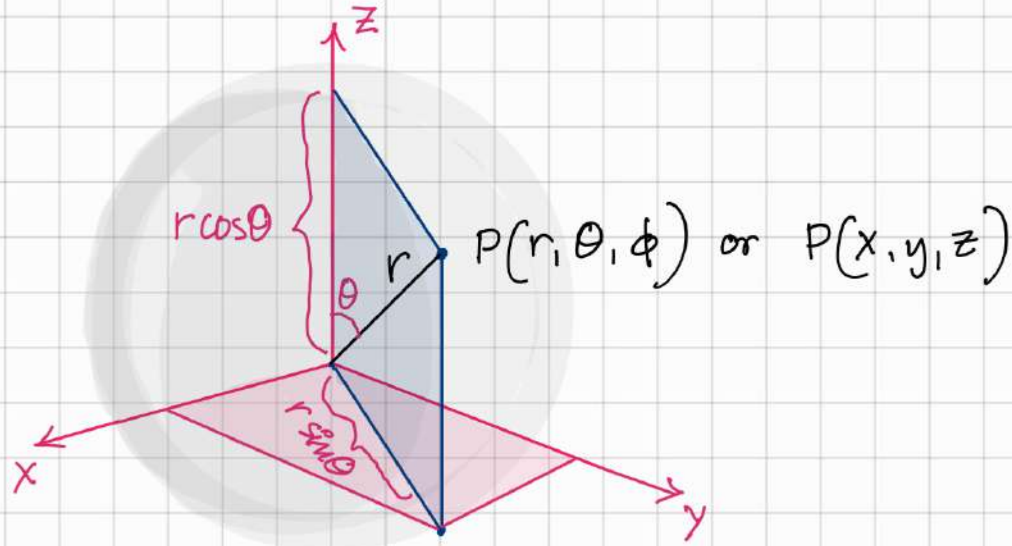
Since ϕ is measured along the X-Y plane, so the maximum value ϕ can assume is
2π or $360°$

Let's find out the relation between r, θ, ϕ and x, y, z mathematically ...

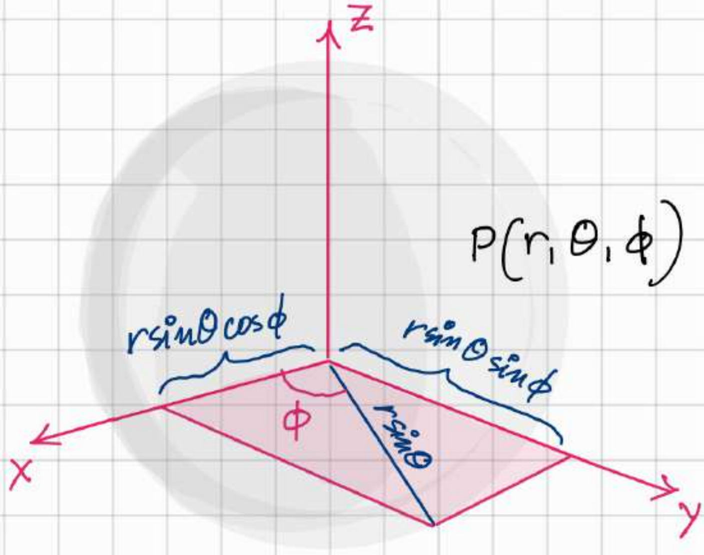


$P(r, \theta, \phi)$ or $P(x, y, z)$

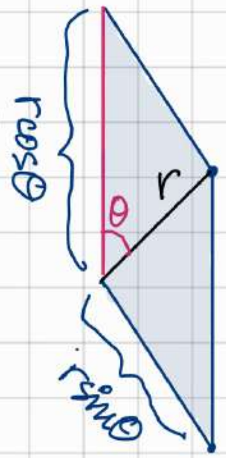
$$\boxed{r = \sqrt{x^2 + y^2 + z^2}}$$



$$\boxed{z = r\cos\theta}$$



P(r, θ, ϕ) or P(x, y, z)



$$\boxed{x = r\sin\theta\cos\phi}$$

$$\boxed{y = r\sin\theta\sin\phi}$$

Summary :

1. For spherical polar co-ordinate system –

   co-ordinates are r, θ and ϕ

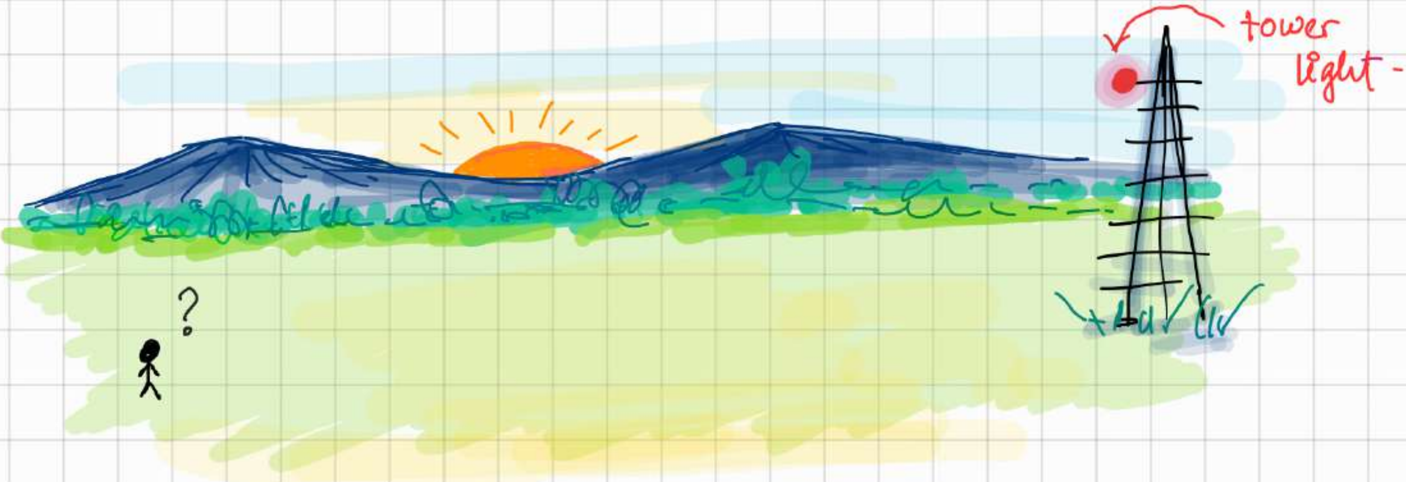
2. Limits ,
   $r : 0 \to \infty$
   $\theta : 0 \to \pi$
   $\phi : 0 \to 2\pi$

3. Relation with the Cartesian Coordinate system –

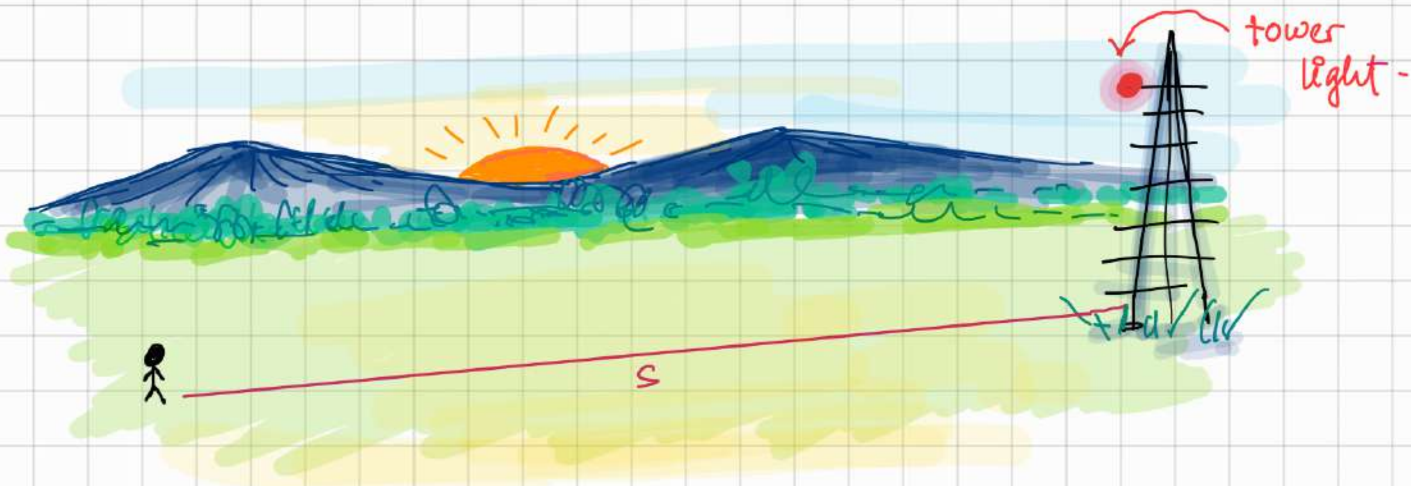
$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

CYLINDRICAL COORDINATE SYSTEM

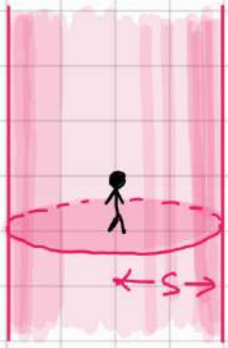
Let's go back to the field. Is there another way we could have described the position of the red light?



First you may measure the distance to the base of the tower along the ground. Let's denote this distance as s.

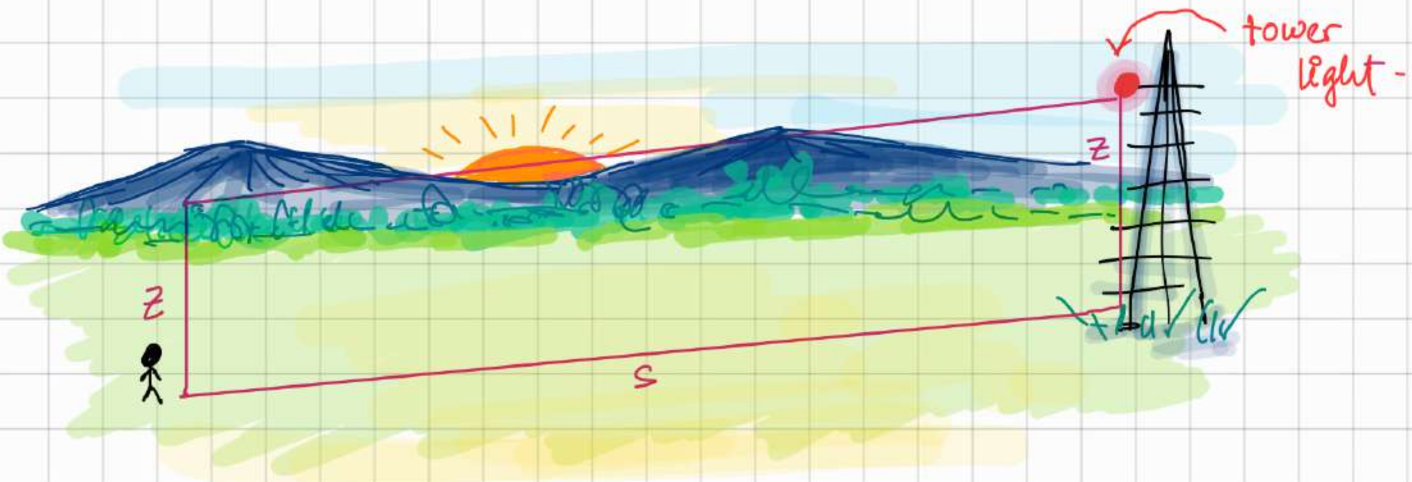


Defining this coordinate effectively means that you have narrowed the position of the object to the curved surface of an infinitely long cylinder with radius s.

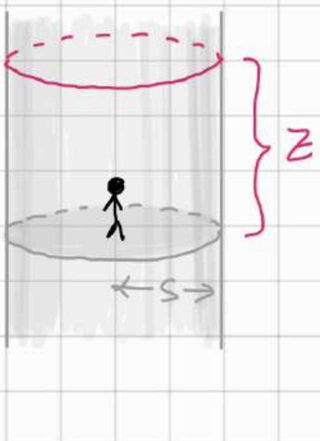


Next you may choose to specify the altitude (or height) at which the light is located.

This is nothing but the familiar z-coordinate.



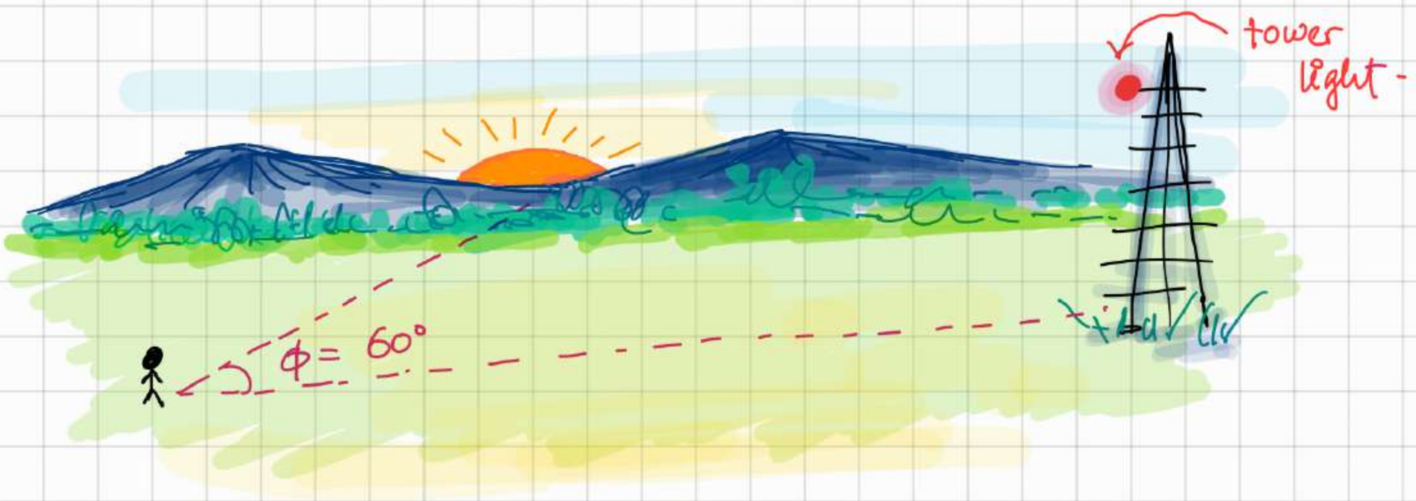
You now know that the light is somewhere on the circumference of a circle with radius s that is raised to a height z above the ground.



What do you do next?

↳ You just specify ϕ.

You look at the rising sun and label this direction as $\phi = 0°$. You measure the angle on the ground from the direction of the rising sun (i.e east) to the light on the tower.



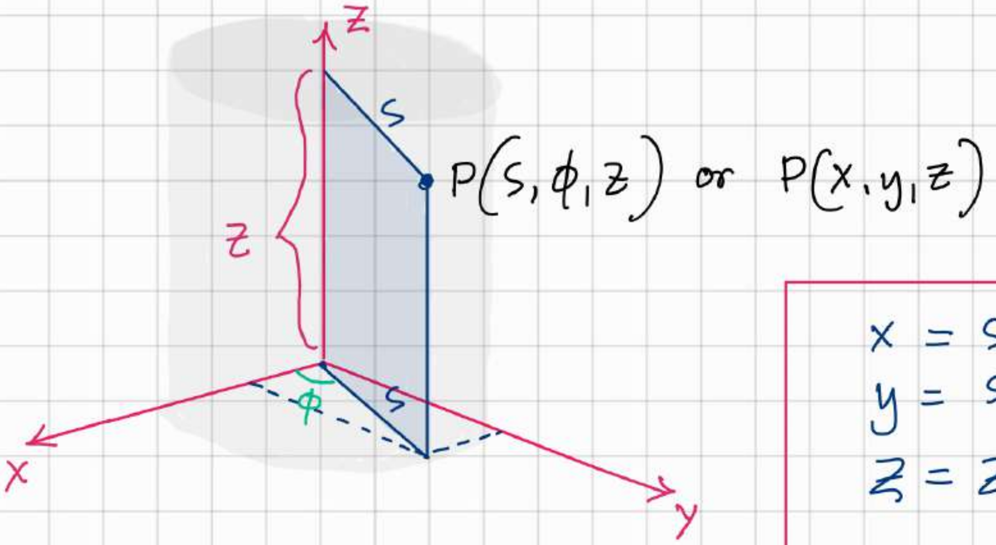
By specifying this direction you have just completed the full description of the position of the light relative to you (you are at the origin)

This is the cylindrical co-ordinate system.

It has the co-ordinates s, ϕ and z

Can you now give me the relation of s, ϕ, z with the cartesian co-ordinates?

Well, 'z' in the cylindrical co-ordinate system is just the same 'z' in the cartesian coordinate system.



$$\boxed{\begin{aligned} x &= s\cos\phi \\ y &= s\sin\phi \\ z &= z \end{aligned}}$$

Summary :

1. For cylindrical co-ordinate system –

   co-ordinates are s, ϕ and z

2. Limits,

$$s : 0 \to \infty$$
$$\phi : 0 \to 2\pi$$
$$z : -\infty \to +\infty$$

3. Relation with the Cartesian Coordinate system –

$$x = s\cos\phi$$
$$y = s\sin\phi$$
$$z = z$$

This was the introduction to spherical polar and cylindrical coordinate systems. In the next few classes you will learn —

(i) unit vectors in these two coordinate systems

(ii) line integral, area integral and volume integral in these coordinate systems.

(iii) Gradient, Divergence, Curl and Laplacian expressions in spherical polar and cylindrical coordinate systems