BINOMIAL DISTRIBUTION (FOR DISCRETE CASE)

Let's say we toss a coin 10 times.

How is the probability of getting heads distributed?

$$P(X = x) = {}^{n}C_{x}\, p^{x}(1-p)^{n-x}$$

Well, for each value of no. of heads we are going to get, it has be calculated with that formula.

Let's dig into it a little bit and try to make sense of what each part represents.

The 1st question to ask is what makes a binomial distribution?

Pre-requisites:

* There are two potential outcomes per **trial**
* The probability of **success** (p) is the same across all trials
* The number of trials (n) is fixed
* Each trial is **independent**

Let's check if the following is a binomial distribution :

Studies show colour blindness affects about 8% of men.
A random sample of 10 men is taken.

* Two potential outcomes per trial – either colourblinded or not colourblinded.

* Across all trials, $p = 0.08$

* No. of trials, $n = 10$ (fixed)

* Each trial is independent.

This is indeed a binomial distribution.

Find the probability that:

(a) All 10 men are colour blind
(b) No men are colour blind
(c) Exactly 2 men are colour blind
(d) At least 2 men are colour blind

a) Let's do it intuitively first.

0.08 x 0.08 x 0.08 x 0.08 x 0.08

0.08 x 0.08 x 0.08 x 0.08 x 0.08

$$P(X = 10) = (0.08)^{10} = 1.07 \times 10^{-11}$$

$$^{10}C_{10}\, p^{10} (1-p)^{10-10}$$

$$= 1 \times (0.08)^{10} \times 1$$

$$= 1.07 \times 10^{-11}$$

b) No men are colourblind ?

$$0.92 \times 0.92 \times 0.92 \times 0.92 \times 0.92$$
$$0.92 \times 0.92 \times 0.92 \times 0.92 \times 0.92$$

$$P(X=0) = (0.92)^{10} = 0.4344$$

$$^{10}C_0 \; p^0 (1-p)^{10-0}$$
$$= 1 \times 1 \times (0.92)^{10}$$
$$= 0.4344$$

c) Exactly two men are colourblind ?

$$0.08 \times 0.08 \times 0.92 \times 0.92 \times 0.92$$
$$0.92 \times 0.92 \times 0.92 \times 0.92 \times 0.92$$

$$0.08 \times 0.92 \times 0.08 \times 0.92 \times 0.92$$
$$0.92 \times 0.92 \times 0.92 \times 0.92 \times 0.92$$

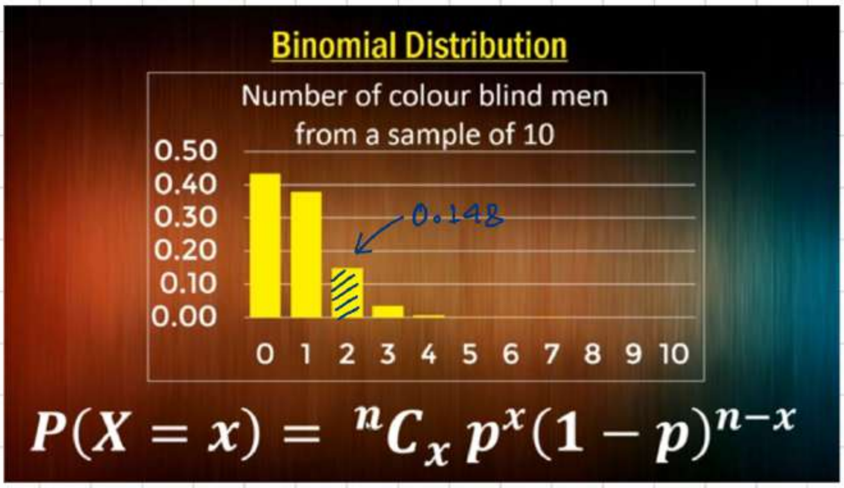
$$0.92 \times 0.92 \times 0.92 \times 0.92 \times 0.92$$
$$0.92 \times 0.92 \times 0.92 \times 0.08 \times 0.08$$

There may be a no. of combinations this can occur.

$^{10}C_2$ term shall take care of it

$$P(X=2) = {}^{10}C_2 \, p^2 (1-p)^{10-2}$$
$$= 45 \times (0.08)^2 \times (0.92)^8$$
$$= 0.148$$

The probability plotted for each outcome is



d) At least 2 men are colourblind ?

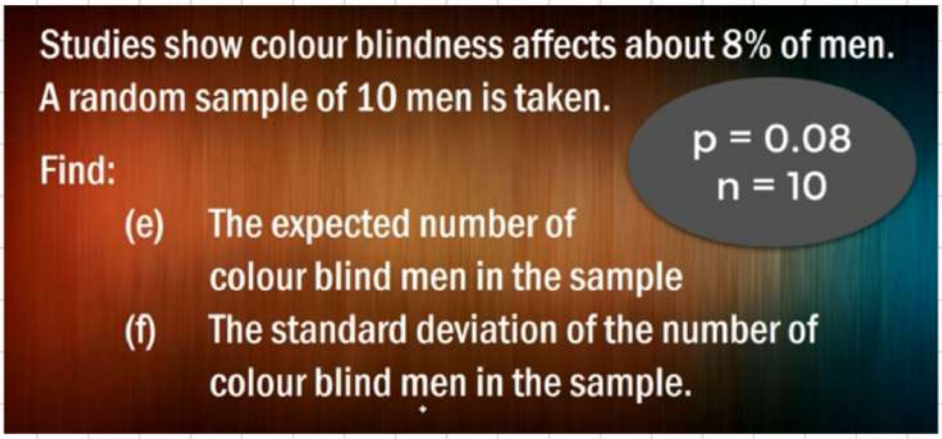
$$P(x \geq 2) = 1 - P(x \leq 1)$$

$$= 1 - P(x=0) - P(x=1)$$

$$= 1 - 0.378 - 0.434$$

$$= 0.188$$

The final thing to do with the binomial distribution is to find the mean and the standard deviation



e) Expected no. of colour blind men

$$E(x) = np = (\text{no. of trials}) \times (\text{prob. of success})$$

$$= 10 \times 0.08 = 0.8$$

f) $$Var(x) = npq = np(1-p) = 10 \times 0.08 \times 0.92 = 0.736$$

$$\text{Std. dev}(x) = \sqrt{npq} = \sqrt{0.736} = 0.858$$