POISSON DISTRIBUTION

QUICK RUNDOWN

* Discrete distribution
* Describes the number of events occuring in a fixed time interval or region of opportunity
* Requires only one parameter, λ
* Bounded by 0 and ∞

Unlike Binomial Distribution where the no. of trials is fixed, Poisson distribution trials continue forever (bounded by 0 and ∞.)

Another feature is that it requires only one parameter λ unlike Binomial dist. which required 2 (n, p).

ASSUMPTIONS

* The rate at which events occur is constant
* The occurrence of one event does not affect the occurrence of a subsequent event (ie. events are independent)

The no. of events occurring within a particular time interval is the same as that occurring at any time interval of equal length.

λ is the mean or expected no. of events within unit time interval.

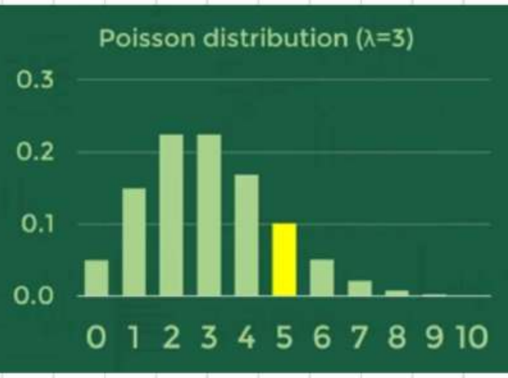
For the Poisson distribution, the probability mass function is given by :

P.M.F:

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P(X = 5) = \frac{e^{-3}3^{5}}{5!}$$

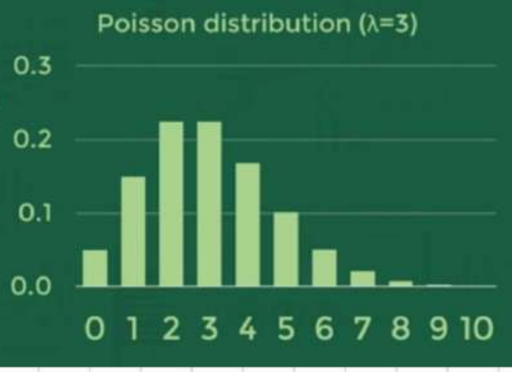
$= 0.101$



But what does the cumulative distribution function looks like ?

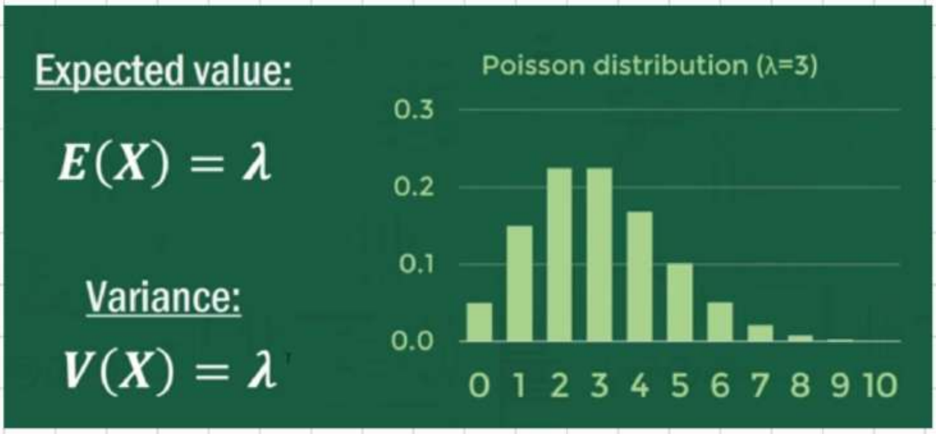
C.D.F:

$$P(X \leq x) = \frac{\Gamma(\lfloor x + 1 \rfloor, \lambda)}{\lfloor x! \rfloor}$$



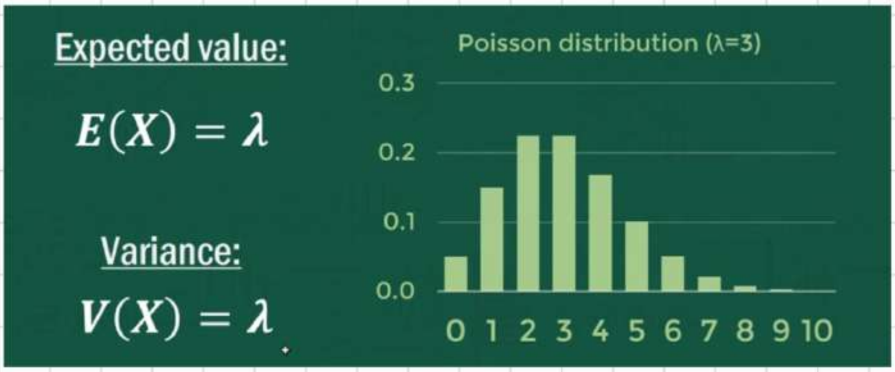
There is a formula for this involving gamma function to calculate the cumulative distribution function.

What about the mean and the std. deviation ?



Mean is just the distribution parameter λ.

Poisson distribution is that special distribution where the variance is just the mean λ.

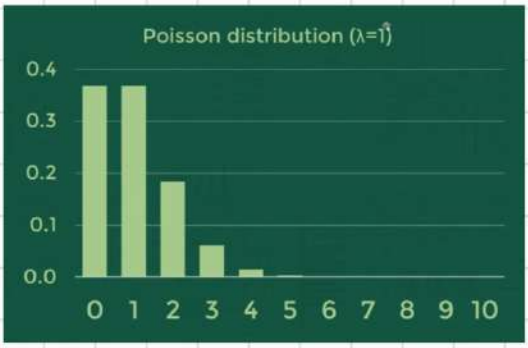


Mean $= \lambda$

Variance $= \lambda$

Std. dev. $= \sqrt{\lambda}$

The Poisson distribution we have shown is for $\lambda = 3$. λ can take any (+)ve real no. values. What do the different distributions look like?

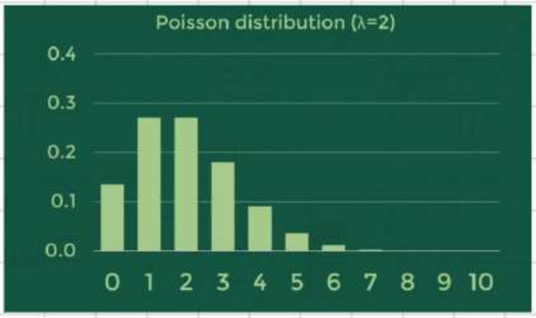


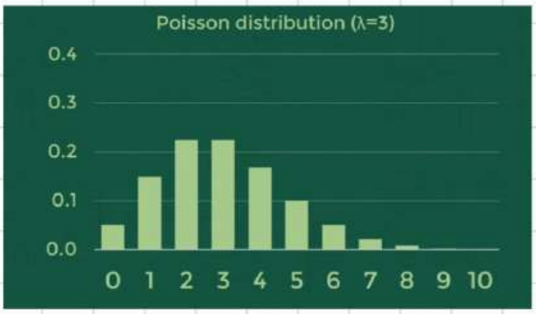
This is quite intuitive.

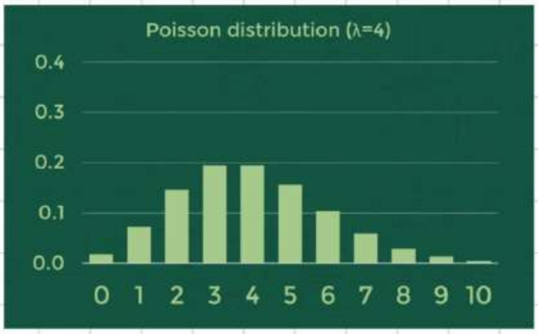
For any event that happens at a constant rate, it is expected that for a mean of 1, the distribution will have a high peak around 1.

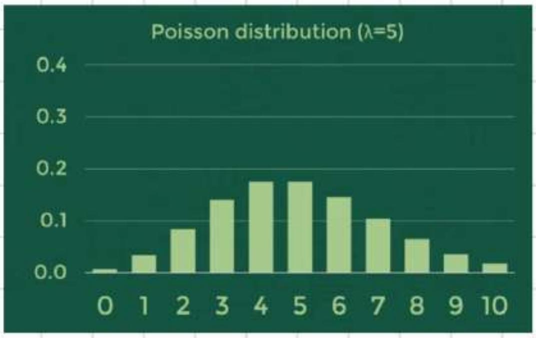
For a mean of 5, the peak at 5 will be less than that at 1 since the distribution would spread out more.

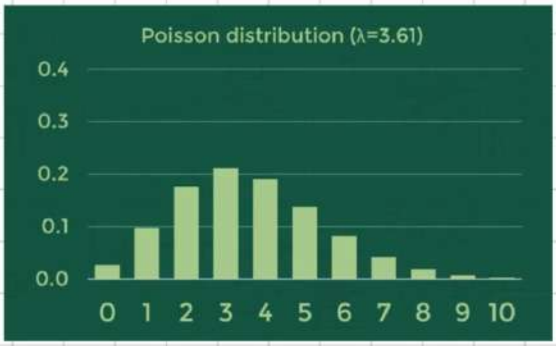
λ doesn't have to be an integer value.

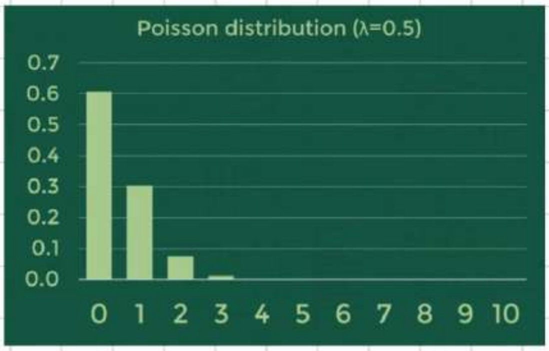












Let's do an example :

Exclusive Vines import Argentinian wine into Australia. They've begun advertising on Facebook to direct traffic to their website where customers can order wine online. The number of click-through sales from the ad is Poisson distributed with a mean of 12 click-through sales per day.

Find the probability of getting:

(a) Exactly 10 click-through sales in the first day

(b) At least 10 click-through sales in the first day

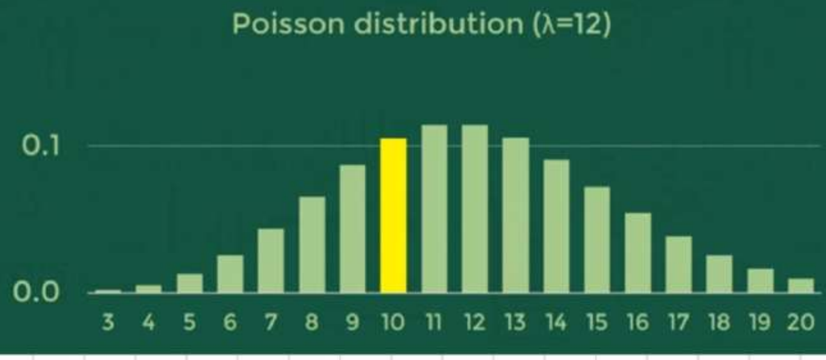
(c) More than one sale in the first hour

BONUS QUESTION:

(d) do you think the poisson distribution is appropriate for this scenario in reality?

Solution :

(a) P(10 click-through sales in the first day) =



$$P(X = 10) = \frac{e^{-12} 12^{10}}{10!}$$

$$= 0.105$$

(b) P(At least 10 click-through sales on the first day) =

$$P(X \geq 10) = ???$$

What we can do is

$$1 - P(x \leqslant 9)$$

$$= 1 - [P(x=0) + P(x=1) + \dots + P(x=9)]$$

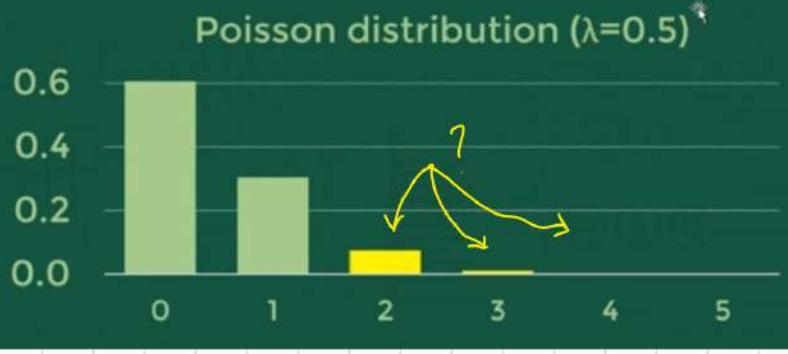
$$= 0.758$$

(c) P(More than 1 click-through sale in the first hour) =

* Calculate the mean for 1 hour

$$\lambda = \frac{12}{24} = 0.5 \ sales\ per\ hour$$

* What is the poisson dist. for $\lambda = 0.5$?



$$1 - [P(x=0) + P(x=1)] = 0.090$$

(d) do you think the Poisson distribution is appropriate for this scenario in reality?

* The rate at which events occur must be constant

(No interval can be more likely to have an event than any other interval of the same size)

Rate of click-through sales can not happen at a constant rate.