Learning Objectives

→ Plane and Spherical Waves

→ Longitudinal and Transverse Waves

→ Plane Progressive (Travelling) Waves

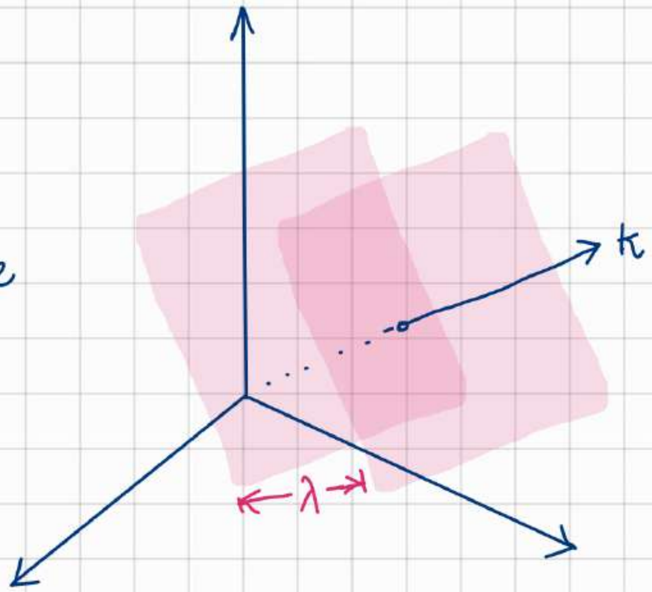
→ Wave Equation

→ Particle and Wave Velocities

→ Pressure of Longitudinal Wave

Plane and Spherical Waves

A plane wave is defined as a wave in which the wave amplitude is constant over all points of a plane perpendicular to the direction of propagation.



'k' is the propagation vector

The equation of the plane wave may be represented as

$$\psi(\vec{r},t) = A e^{i(\vec{k}\cdot\vec{r} \mp \omega t)}$$

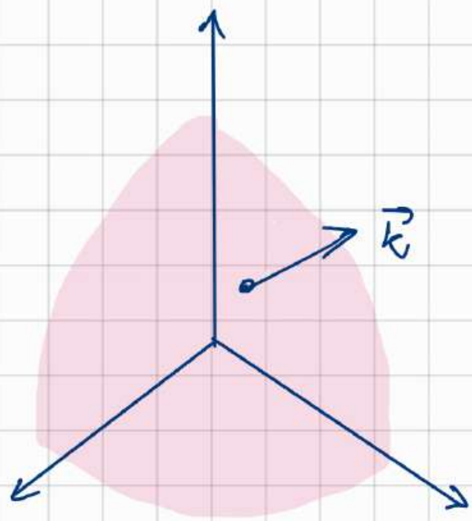
$A \rightarrow$ amplitude
$\omega \rightarrow$ angular frequency

For a plane wave, the disturbance travels along the k-direction, and a phase corresponds to it at each point in space and time.

At any given time, the surfaces joining all points of equal phase are known as wavefronts.

A spherical wave is a wave whose surfaces of common phase are spheres and the source of waves is a central point.

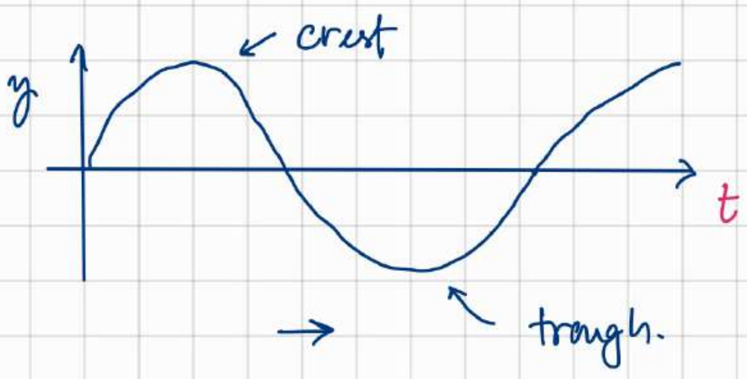
$$\psi(\vec{r},t) = \left(\frac{A}{r}\right) e^{ik(r \mp vt)}$$



Transverse and Longitudinal Wave

* Transverse Waves

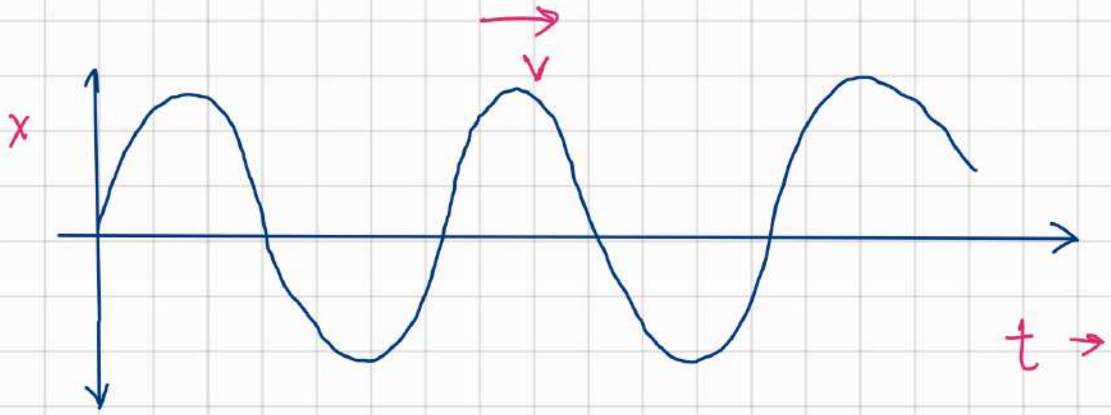
Oscillations of the particles in the propagating medium are displaced perpendicular to the direction of propagation of the wave.



$$\vec{s} = A \sin(kx - \omega t)\,\hat{y}$$

* Longitudinal Wave.

Oscillations of the particles in the propagating medium vibrate along the direction of propagation of the wave.



$$\vec{s} = A \sin(kx - \omega t)\,\hat{x}$$

Plane Progressive Waves or Travelling Waves.

1. These waves propagate in the forward direction of the medium with a finite velocity.

2. In progressive waves, energy and momentum are transfered outwards from the wave source and the wave profile moves in the wave's propogation direction.

3. In progressive waves, pressure and density have equal changes at all medium points.

4. When the wave passes through, the medium do not move; rather, the particles of the medium vibrate around a mean position. The net displacement of the particle is zero.

Distinction between Travelling Wave and Stationary Wave

1. Travelling wave is an advancing wave which moves in the medium continuously with a finite velocity but for stationary waves, there is no advancement of the wave in any direction.

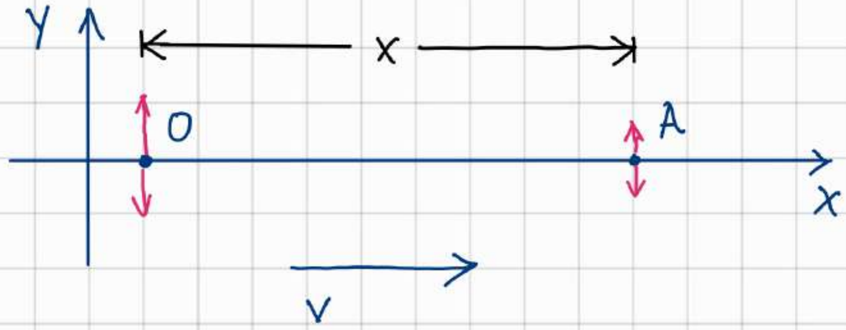
2. Energy flows across every plane in the direction of propagation of the travelling wave whereas there is no flow of energy across any plane for stationary waves.

3. For travelling waves no particle of the propagating medium is permanently at rest – each particle of the medium execute simple harmonic motion about its mean position with same amplitude.

However, for standing waves, nodes are permanently at rest. Except nodes, all other particles execute simple harmonic motion with varying amplitudes.

4. For travelling waves, the phase of vibration varies continuously from point to point. For stationary waves, all the points between any pair of nodes vibrate in the same phase, but the phase suddenly reverses at each node.

Mathematical representation of a plane progressive wave



Suppose a wave is propagating from left to right with velocity v.

O and A are two points situated x dist. apart in the medium whose phase lags by a phase difference α at any instant of time t.

Since particles at O and A undergo simple harmonic motion about their mean position, so their motion along the Y-direction may be represented as –

At O, $y = a \sin(\omega t)$

At A, $y = a \sin(\omega t - \alpha)$

* a is the amplitude
α is the phase difference

We know, for a phase difference of 2π, the path difference is λ.

According to our assumption, a path difference of x corresponds to a phase difference α.

Thus, $$\frac{\alpha}{x} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \alpha = \frac{2\pi x}{\lambda}$$

Also, $\omega = \frac{2\pi}{T} = 2\pi f$

$= 2\pi \frac{v}{\lambda}$

Thus, the equation of the particle at A is

$$y = a \sin\left(\frac{2\pi v}{\lambda} t - \frac{2\pi x}{\lambda}\right)$$

$$y = a \sin\left(\frac{2\pi}{\lambda}[vt - x]\right)$$

This equation describes a harmonic wave.

We may also write this equation as –

$$y = a \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)$$

$$= a \sin(\omega t - kx)$$

For a wave travelling from right to left,

$$y = a \sin\left(\frac{2\pi}{\lambda}[vt + x]\right)$$

Differential Equation of Wave Motion.

$$y = a \sin\left(\frac{2\pi}{\lambda}[vt - x]\right)$$

$$\frac{\partial y}{\partial t} = a \frac{2\pi v}{\lambda} \cos\left(\frac{2\pi}{\lambda}[vt - x]\right)$$

$$\frac{\partial^2 y}{\partial t^2} = -a \frac{4\pi^2 v^2}{\lambda^2} \sin\left(\frac{2\pi}{\lambda}[vt - x]\right) \rightarrow (1)$$

$$\frac{\partial y}{\partial x} = -a \frac{2\pi}{\lambda} \cos\left(\frac{2\pi}{\lambda}[vt - x]\right)$$

$$\frac{\partial^2 y}{\partial x^2} = -a \frac{4\pi^2}{\lambda^2} \sin\left(\frac{2\pi}{\lambda}[vt - x]\right)$$

or $$v^2 \frac{\partial^2 y}{\partial x^2} = -a \frac{4\pi^2 v^2}{\lambda^2} \sin\left(\frac{2\pi}{\lambda}[vt - x]\right) \rightarrow (2)$$

We see that

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

↳ This is the wave equation.

Relation between Particle Velocity and Wave Velocity.

Particle velocity is the velocity of the particles oscillating about the mean position.

$$y = a \sin\left(\frac{2\pi}{\lambda}[vt - x]\right)$$

$\therefore$ particle velocity, $v_p = \frac{dy}{dt}$

$$= a\,\frac{2\pi v}{\lambda} \cos\left(\frac{2\pi}{\lambda}[vt - x]\right)$$

maximum velocity $= a\,\frac{2\pi v}{\lambda}$

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particle acceleration, $A_p = \frac{d^2y}{dt^2}$

$$= -\frac{4\pi^2 v^2}{\lambda^2}\left[a \sin\left(\frac{2\pi}{\lambda}[vt-x]\right)\right]$$

$$= -\frac{4\pi^2 v^2}{\lambda^2}\, y.$$

Thus, acceleration of the particle is maximum when $y = a$ i.e. at maximum amplitude points.

Significance of the negative sign ?

↳ the negative sign implies that the acceleration is directed towards the mean position.

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Now $\dfrac{dy}{dx} = -a \dfrac{2\pi v}{\lambda} \cos\left(\dfrac{2\pi}{\lambda}[vt - x]\right)$

$$\therefore \frac{dy}{dt} = -v \frac{dy}{dx}.$$

The relation between particle velocity and wave velocity is

$$\left[\begin{array}{c}\text{particle velocity at} \\ \text{any instant}\end{array}\right] = -\left[\text{Wave velocity}\right] \times \left[\begin{array}{c}\text{slope of displacement} \\ \text{curve at that} \\ \text{instant}\end{array}\right]$$

Pressure Distribution for a plane progressive wave.

$$y = a \sin\left(\frac{2\pi}{\lambda}[vt - x]\right)$$

$$\text{particle velocity} = \frac{dy}{dx} = a\frac{2\pi v}{\lambda}\cos\left(\frac{2\pi}{\lambda}[vt-x]\right)$$

What is the strain in the medium?

$\hookrightarrow \frac{dy}{dx}$

Bulk modulus, $K = \dfrac{\text{change in pressure}}{\text{strain}}$

$$K = \frac{-dP}{dy/dx}$$

$$\Rightarrow dP = -K\frac{dy}{dx}$$

$$\Rightarrow dP = K\left(-\frac{dy}{dx}\right)$$

* If $\frac{dy}{dx}$ is negative, dP is (+)ve $\rightarrow$ compression
* If $\frac{dy}{dx}$ is positive, dP is (−)ve $\rightarrow$ rarefaction.