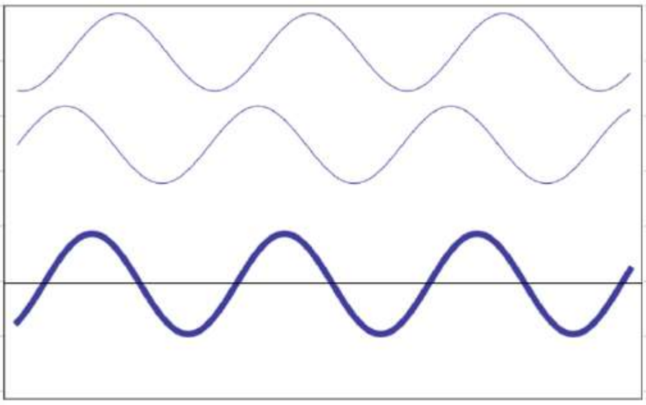
Superposition of Waves

Just like superposition of simple harmonic oscillators, the principle of superposition may be applied to waves whenever two (or more) waves travelling through the same medium at the same time.

The waves pass through each other without being disturbed.

The net displacement of the medium at any point in space and time, is simply the sum of the individual wave displacements. This is true of waves which are finite in length (wave pulses) or which are continuous sine waves.

Constructive and Destructive Interference



Let us suppose, two waves with the same amplitude, frequency and wavelength are travelling in the same direction.

$$y_1(x,t) = a \sin(kx - \omega t)$$

$$y_2(x,t) = a \sin(kx - \omega t + \phi)$$

The resultant wave displacement may be written as

$$y = y_1 + y_2$$
$$= a \sin(kx - \omega t) + a \sin(kx - \omega t + \phi)$$
$$= 2a \cos\left[\frac{(kx-\omega t+\phi)-(kx-\omega t)}{2}\right] \cdot \sin\left[\frac{(kx-\omega t+\phi)+(kx-\omega t)}{2}\right]$$
$$= 2a \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

The amplitude depends on the phase ϕ.

It is maximum when $\cos\left(\frac{\phi}{2}\right)$ is ± 1

i.e. $\frac{\phi}{2} = n\pi \quad [n = 0, 1, 2, \ldots]$

$\Rightarrow \phi = 2n\pi$

This is known as constructive interference

The amplitude is minimum when $\cos\left(\frac{\phi}{2}\right) = 0$

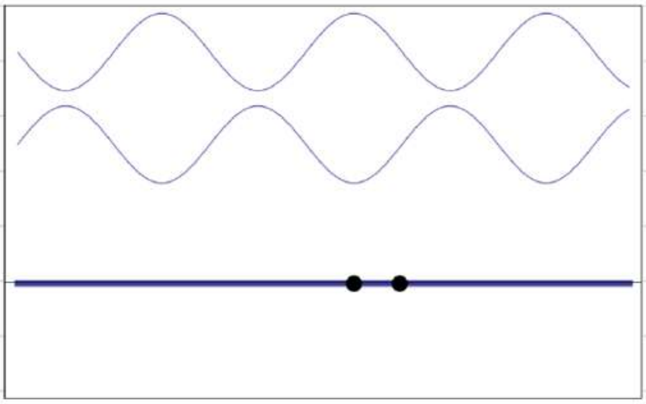
i.e. $\frac{\phi}{2} = (2n+1)\frac{\phi}{2} \quad [n = 0, 1, 2, \ldots]$

$\Rightarrow \phi = (2n+1)\phi$

This is destructive interference.

Standing Waves

Two sine waves moving in opposite directions create a standing wave.



Let the two sine waves travelling in opposite directions be represented by

$$y_1 = a \sin(kx - \omega t)$$

$$y_2 = a \sin(kx + \omega t)$$

The resultant amplitude is

$$y = y_1 + y_2$$

$$= a \sin(kx - \omega t) + a \sin(kx + \omega t)$$

$$= 2a \sin\left[\frac{(kx-\omega t)+(kx+\omega t)}{2}\right] \cos\left[\frac{(kx-\omega t)-(kx+\omega t)}{2}\right]$$

$$= 2a \sin(kx) \cos(\omega t)$$

This wave is no longer a travelling wave because the position and time dependence have been separated.

The wave amplitude is a function of position

$$2a \sin(kx)$$

This amplitude does not travel, but stands still and oscillate up and down according to $\cos(\omega t)$

Points of maximum displacement are called antinodes and points of zero displacement are called nodes.

Beats

Two waves of equal amplitude travelling in the same direction with slightly different frequencies travelling with the same velocity give rise to beats.

$$y_1 = a \sin(k_1 x - \omega_1 t)$$
$$y_2 = a \sin(k_1 x - \omega_2 t)$$

$$y = y_1 + y_2$$
$$= a \sin(k_1 x - \omega_1 t) + a \sin(k_2 x - \omega_2 t)$$
$$= 2a \cos\left(\frac{k_1 - k_2}{2} x - \frac{\omega_1 - \omega_2}{2} t\right) \sin\left(\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t\right)$$

The resulting particle motion is the product of two travelling waves.

One part is a sine wave which oscillates with the average frequency

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

This is the frequency which is perceived by the listner.

The other part is a cosine wave which oscillates with the frequency.

$$\omega = \frac{\omega_1 - \omega_2}{2}$$

This term controls the amplitude "envelope" of the wave and causes the perception of beats.