1. Prove $\mathcal{L}[t^n] = \dfrac{\Gamma(n+1)}{s^{n+1}}$ if $n > -1$, $s > 0$

if $\Gamma(n)$ is defined as $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$

$$\mathcal{L}[t^n] = \int_0^\infty e^{-st} t^n dt$$

Let $st = x$ $(s > 0)$

then

$$\mathcal{L}[t^n] = \int_0^\infty e^{-x} \left(\frac{x}{s}\right)^n d\left(\frac{x}{s}\right)$$

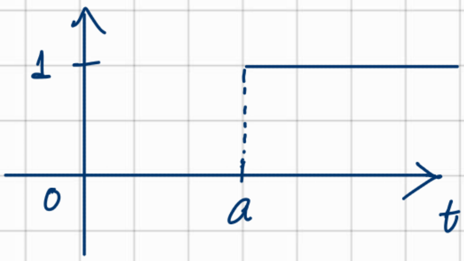
$$= \frac{1}{s^{n+1}} \int_0^\infty e^{-x} x^{(n+1)-1} dx$$

$$= \frac{\Gamma(n+1)}{s^{n+1}}$$

2. What is the Laplace Transform of the Heavyside Step Function?

The Heavyside or Unit Step Function is defined as -

$$f(t) = \begin{cases} 0 & \text{for } t < a ; \\ 1 & \text{for } t \geqslant a \end{cases}$$



The Laplace Transform of the Heavyside function is.

$$\mathcal{L}\left[f(t)\right] = \int_0^{\infty} f(t) e^{-st} \, dt$$

$$= \int_a^{\infty} 1 . e^{-st} \, dt$$

$$= \frac{e^{-at}}{s}$$

3: Find the Laplace Transform of the Dirac Delta Function.

The Dirac Delta Function is defined as -

$$\delta(t-a) = \begin{cases} \infty & \text{for } t = a \\ 0 & \text{for } t \neq a. \end{cases}$$

The Dirac Delta Function follows the property -

$$\int_{-\infty}^{+\infty} f(t)\,\delta(t-a) = f(a).$$

Thus, the Laplace tramsfrom of the Dirac Delta Function $\delta(t-a)$ for $a > 0$ is -

$$\mathcal{L}\left[\delta(t-a)\right] = \int_{0}^{\infty} \delta(t-a)\,e^{-st}\,dt$$

$$= e^{-as}.$$