Q:- Prove the following :-

a) $\delta_{ii} = 3$

$$LHS = \delta_{ii}$$
$$= \delta_{11} + \delta_{22} + \delta_{33}$$
$$= 1 + 1 + 1$$
$$= 3$$
$$= RHS$$

b) $\delta_{ik}\,\varepsilon_{ikm} = 0$

We know that the Levi-Civita tensor is antisymmetric with respect to swapping two indices.

i.e. $\varepsilon_{ikm} = -\varepsilon_{kim}$

Multiplying by δ_{ik} on both sides,

$$\delta_{ik}\,\varepsilon_{ikm} = -\delta_{ik}\,\varepsilon_{kim}$$

Since, δ_{ik} is symmetric w.r.t. indices i & k.

$$\therefore \delta_{ik}\,\varepsilon_{ikm} = -\delta_{ki}\,\varepsilon_{kim}$$

Hence, the RHS may be written as :-

$$\delta_{ik}\,\varepsilon_{ikm} = -\delta_{ik}\,\varepsilon_{ikm}$$

$$\Rightarrow \delta_{ik}\,\varepsilon_{ikm} = 0$$

Q: Prove that

$$\vec{A} \circ \vec{B} = \sum_{i=1}^{3} \sum_{j=1}^{3} A_i B_i \delta_{ij}$$

$$\vec{A} = A_1 \hat{e}_1 + A_2 \hat{e}_2 + A_3 \hat{e}_3 = \sum_{i=1}^{3} A_i \hat{e}_i$$

$$\vec{B} = B_1 \hat{e}_1 + B_2 \hat{e}_2 + B_3 \hat{e}_3 = \sum_{j=1}^{3} B_j \hat{e}_j$$

considering $\hat{e}_i$ to be an orthonormal basis,

$$\hat{e}_1 \circ \hat{e}_1 = \hat{e}_2 \circ \hat{e}_2 = \hat{e}_3 \cdot \hat{e}_3 = 1$$

and $\hat{e}_1 \cdot \hat{e}_2 = \hat{e}_2 \cdot \hat{e}_1 = 0$

$$\hat{e}_2 \cdot \hat{e}_3 = \hat{e}_3 \cdot \hat{e}_2 = 0$$

$$\hat{e}_3 \cdot \hat{e}_1 = \hat{e}_1 \cdot \hat{e}_3 = 0$$

This allows us to write

$$\hat{e}_i \cdot \hat{e}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} = \delta_{ij}$$

$$\therefore \vec{A} \cdot \vec{B} = \left(\sum_{i=1}^{3} A_i \hat{e}_i \right) \cdot \left(\sum_{j=1}^{3} B_j \hat{e}_j \right)$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_i B_j \left(\hat{e}_i \cdot \hat{e}_j \right)$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_i B_j \delta_{ij}$$

Q:- Obtain the Fourier Sine and Cosine Transform of the function -

$$f(x) = \begin{cases} 1 & ; \; 0 < x < \pi/2 \\ 0 & ; \; x > \pi/2 \end{cases}$$

Fourier Sine Transform

$$F(k) = -i\sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\sin(kx)\,dx$$

$$= -i\sqrt{\frac{2}{\pi}} \left\{ \int_0^{\pi/2} 1 \cdot \sin(kx)\,dx + \int_{\pi/2}^\infty 0 \cdot \sin(kx)\,dx \right.$$

$$= -i\sqrt{\frac{2}{\pi}} \int_0^{\pi/2} \sin(kx)\,dx$$

$$= \; ?$$

Fourier Cosine Transform

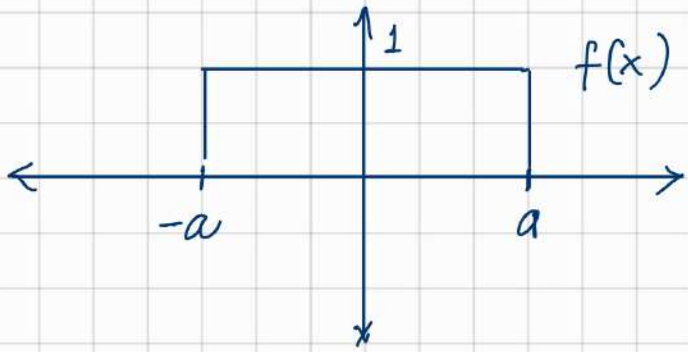
$$F(k) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\cos(kx)\,dx$$

$$= \sqrt{\frac{2}{\pi}} \left\{ \int_0^{\pi/2} 1 \cdot \cos(kx)\,dx + \int_{\pi/2}^\infty 0 \cdot \cos(kx)\,dx \right\}$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\pi/2} \cos(kx)\,dx \quad = \; ?$$

Q: Find the Fourier Transform of the function -

$$f(x) = \begin{cases} 1 \ , & \text{for } |x| \leq a \\ 0 \ ; & \text{for } |x| > a \end{cases}$$



$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \int_{-\infty}^{-a} 0 \cdot e^{-ikx} dx + \int_{-a}^{+a} 1 \cdot e^{-ikx} dx + \int_{a}^{0} 0 \cdot e^{-ikx} dx \right\}$$

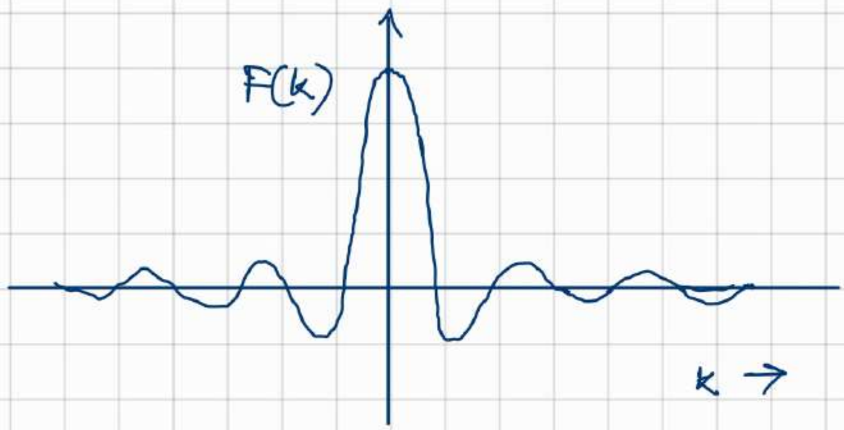
$$= \frac{1}{\sqrt{2\pi}} \left\{ \int_{-a}^{+a} e^{-ikx} dx \right\}$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \left. \frac{e^{-ikx}}{-ik} \right|_{-a}^{+a} \right\}$$

$$= \frac{1}{\sqrt{2\pi}} \left(-\frac{1}{ik} \right) \left\{ e^{-iak} - e^{iak} \right\}$$

$$= \frac{2}{\sqrt{2\pi}} \cdot \frac{1}{k} \left\{ \frac{e^{iak} - e^{-iak}}{2i} \right\}$$

$$= \sqrt{\frac{2}{\pi}}\ \frac{\sin(ak)}{k}$$

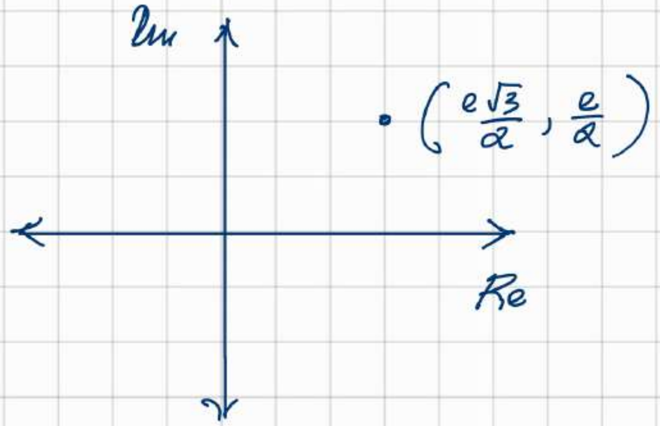


Q: $e^{\left(1-\pi/6\, i\right)}$

$$e^{1} \cdot e^{i\pi/6}$$

$$e^{1} \cdot \left\{ \cos\left(\pi/6\right) + i\sin\left(\pi/6\right) \right\}$$

$$e \cdot \left\{ \frac{\sqrt{3}}{2} + \frac{i}{2} \right\}$$



Associated Tensors.

$$A^p e_p \longleftrightarrow A_p \epsilon^p$$

$$e_p e_q = g_{pq} \qquad \epsilon^p \epsilon^q = h^{pq}$$

$$e_p \epsilon^q = \delta_p^q \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$g_{pq} \xleftrightarrow{inv} g^{pq}$$

$$\hat{i}\cdot\hat{i} = \quad \hat{i}\cdot\hat{j} \quad \hat{j}\cdot\hat{i}$$

$$g_{pq} A^q = A_p \quad ; \quad g_{qp} A^p = A_q$$

$$g_{pq} = g_{qp}$$

$$T^a{}_{bc} = T^a T_b T_c \qquad g_{ap} T^a T_b T_c = T_p T_{bc}$$

$$= T_{pbc}$$

COUNTING UNIQUE ELEMENTS OF MATRICES.

(i) Symmetric Matrix : $A_{n \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

<u>For off-diagonal entries</u>

$$a_{ij} = a_{ji} \quad \text{for} \quad i \neq j$$

Thus, unique off diagonal entries.

$= \frac{1}{2}$ (total diagonal entries.).

Now, total off-diagonal entries = total matrix entries − no. of diagonal entries

$= n^2 - n$

$\therefore$ unique off-diagonal entries $= \dfrac{n^2 - n}{2}$

<u>For diagonal entries.</u>

there are n unique diagonal entries a_{ii}

---

total unique entries = unique off-diagonal entries + unique diagonal entries

$$= \frac{n^2 - n}{2} + n = \frac{n^2 - n + 2n}{2} = \frac{n^2 + n}{2} = \frac{n(n+1)}{2}$$

(ii) Skew- Symmetric Matrix

$$\begin{bmatrix} 0 & a_{12} & a_{13} \\ -a_{12} & 0 & a_{23} \\ -a_{13} & -a_{23} & 0 \end{bmatrix}$$

For off-diagonal entries

$$a_{ij} = -a_{ji} \quad \text{for} \quad i \neq j$$

Thus, unique off diagonal entries.

$= \frac{1}{2}$ (total diagonal entries.).

Now, total off-diagonal entries = total matrix entries − no. of diagonal entries

$= n^2 - n$

$\therefore$ unique off-diagonal entries $= \frac{n^2-n}{2}$

For diagonal entries.

there are 0 unique diagonal entries $(a_{ii} = 0)$

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total unique entries = unique off-diagonal entries + unique diagonal entries

$$= \frac{n^2-n}{2} + 0 = \frac{n(n-1)}{2}$$