

ORTHOGONAL COORDINATE SYSTEM

	h_1	h_2	h_3
Cartesian System (x, y, z)	1	1	1
Spherical Polar System (r, θ, ϕ)	1	r	$r \sin \theta$
Cylindrical System (s, ϕ, z)	1	s	1

GRADIENT

$$\vec{\nabla} f = \frac{1}{h_1} \frac{\partial f}{\partial q_1} \hat{q}_1 + \frac{1}{h_2} \frac{\partial f}{\partial q_2} \hat{q}_2 + \frac{1}{h_3} \frac{\partial f}{\partial q_3} \hat{q}_3$$

Cartesian Co-ordinate System (x, y, z)

$$\vec{\nabla} f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$$

Spherical Polar Co-ordinate System (r, θ, ϕ)

$$\vec{\nabla} f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}$$

Cylindrical Co-ordinate System (s, ϕ, z)

$$\vec{\nabla} f = \frac{\partial f}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

DIVERGENCE

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial q_1} (A_1 h_2 h_3) + \frac{\partial}{\partial q_2} (h_1 A_2 h_3) + \frac{\partial}{\partial q_3} (h_1 h_2 A_3) \right]$$

Cartesian Co-ordinate System (x, y, z)

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Spherical Polar Co-ordinate System (r, θ, ϕ)

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

Cylindrical Coordinate System (s, ϕ, z)

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{s} \frac{\partial}{\partial s} (s A_s) + \frac{1}{s} \frac{\partial}{\partial \phi} (A_\phi) + \frac{\partial}{\partial z} A_z$$

CURL

$$\vec{\nabla} \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} \hat{q}_1 h_1 & \hat{q}_2 h_2 & \hat{q}_3 h_3 \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$$

Cartesian Coordinate System (x, y, z)

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Spherical Polar Co-ordinate System (r, θ, ϕ)

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \frac{\hat{r}}{r^2 \sin \theta} & \frac{\hat{\theta}}{r \sin \theta} & \frac{\hat{\phi}}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

Cylindrical Coordinate System (s, ϕ, z)

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_s & s A_\theta & A_\phi \end{vmatrix}$$

LAPLACIAN

$$\nabla^2 f = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial q_1} \left(\frac{h_2 h_3}{h_1} \frac{\partial f}{\partial q_1} \right) + \frac{\partial}{\partial q_2} \left(\frac{h_3 h_1}{h_2} \frac{\partial f}{\partial q_2} \right) + \frac{\partial}{\partial q_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial f}{\partial q_3} \right) \right]$$

Cartesian Coordinate System (x, y, z)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Polar Coordinate System (r, θ, ϕ)

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

Cylindrical Coordinate System (s, ϕ, z)

$$\nabla^2 f = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial f}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$